MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

TMA1201 – DISCRETE STRUCTURES AND PROBABILITY

(All sections / Groups)

24 FEBRUARY 2017
3.00 pm – 5.00 pm
(2 Hours)

INSTRUCTION TO STUDENT

1. This question paper consists of 7 printed pages (inclusive of the front page) with 5 questions only. Page 5 is an appendix for the rules of inference and logical equivalence laws. Last two pages are the Standard Normal distribution table.
2. Attempt **ALL 5** questions. The distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided.
4. Show **ALL** of your working steps clearly.
5. **NO** calculator is allowed in this examination.

QUESTION 1

a) Let A be set of vowels in the English alphabet and $B = \{x \mid x \text{ is a letter from the word "calculus"}\}$.

i) List out the unique elements of set A and set B. [1 mark]

ii) Find $A \Delta B$. [1 mark]

iii) Find $P(A - B)$. [1 mark]

b) Let $C = \{0, 2, 3\}$, $D = \{4, 5, 9\}$ and the relation $R = \{(x, y) \in C \times D \mid x^2 = y\}$.

i) List all the elements of $C \times D$. [1 mark]

ii) List all the elements of R. [1 mark]

iii) Is R a function? Explain your answer. [1 mark]

c) Suppose S is a relation on Z^+ such that $S = \{(a,b) \mid a \leq b\}$. Is S transitive? Justify your answer. [2 marks]

d) Let $f : Z^+ \to Z^+$ where $f(x) = x^2 + 1$.

i) Find the range of the function f. [1 mark]

ii) Determine whether f is onto. Explain your answer. [1 mark]

QUESTION 2

a) Construct a TRUTH TABLE for the proposition $[\neg p \wedge (p \vee q)] \to q$. [2 marks]

b) Show that the propositions $(p \to \neg q) \wedge (p \to \neg r)$ and $p \to \neg(q \vee r)$ are logically equivalence by using equivalence laws. [2 marks]

c) Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \to \neg S(x))$, and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true. [6 marks]

Continued .....

QUESTION 3

a) Given the recursion

$$r_0 = 1,$$
$$r_1 = 7,$$
$$r_n = 2r_{n-1} + 3r_{n-2}, \quad n \geq 2.$$

i) Find the values of r_2 and r_3. [2 marks]

ii) By using mathematical induction, show that

$$r_n = (-1)^{n+1} + 2 \cdot 3^n \text{ for } n \geq 0.$$ [5 marks]

b) Show that the average-case complexity of $f(x) = 4x^3 + x \ln x$ is $\Theta(x^3)$.

[3 marks]

QUESTION 4

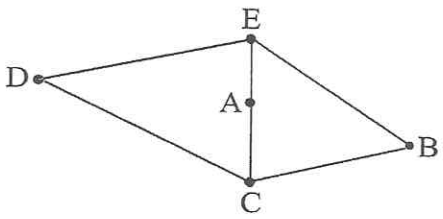
a) Given the following state-transition table of a finite-state automaton M, with S_0 as the initial state and S_1 as the accepting state:

	Next State	
	a	*b*
S_0	S_1	S_0
S_1	S_1	S_2
S_2	S_2	S_1

i) Is M accepting the string *baba*? Explain your answer. [2 marks]

ii) State the language accepted by M. [2 marks]

b) Given the following graph:



i) Is this graph has an Euler cycle? Justify your answer. [2 marks]

ii) Use the breadth first search algorithm to find a spanning tree in the above graph. Assume vertex **A** is the root and the vertices are ordered alphabetically. Show clearly each step of how the algorithm is performed and present your answer in a table with three columns, the first column is the number of steps, the second column is the queue for the search algorithm and the third column is the edges of the resulting spanning tree.

[4 marks]

Continued .....

QUESTION 5

a) Two students will be selected to represent a class in a competition. Find the probability of selecting at most one Indian student if this class has 5 Malay students, 4 Chinese students and 3 Indian students. You should leave your answer in the simplest fraction form. [4 marks]

b) The number of accident per day in a particular area is known to follow a Poisson distribution with mean 0.2. Find the probability of more than 1 accident happen in this area in a period of 5 days. You should leave your answer in the simplest form of mathematical expression. [3 marks]

c) The waiting time for a customer to be served by a bank teller is known to be normally distributed with mean 6 minutes and standard deviation 1 minute. Find the probability that a customer need to wait for at least 5 minutes before he is served by this teller. [3 marks]

End of Page.

Appendix

List of Rules of Inference

ADD : $p \Rightarrow (p \vee q)$

SIMP : $[p \wedge q] \Rightarrow p$

CONJ : $p \wedge q \Rightarrow (p \wedge q)$

MP : $[(p \rightarrow q) \wedge p] \Rightarrow q$

MT : $[(p \rightarrow q) \wedge (\neg q)] \Rightarrow \neg p$

HS : $[(p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow (p \rightarrow r)$

DS : $[(p \vee q) \wedge (\neg p)] \Rightarrow q$

RES : $[(p \vee q) \wedge (\neg p \vee r)] \Rightarrow (q \vee r)$

List of Logical Equivalence Laws

Conversion of Implication: $p \rightarrow q \Leftrightarrow \neg p \vee q$

Double Negation: $\neg\neg p \Leftrightarrow p$

DeMorgan : (i) $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$

(ii) $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$

Domination : (i) $p \wedge F \Leftrightarrow F$ (ii) $p \vee T \Leftrightarrow T$

Negation : (i) $p \wedge \neg p \Leftrightarrow F$ (ii) $p \vee \neg p \Leftrightarrow T$

Identity : (i) $p \wedge T \Leftrightarrow p$ (ii) $p \vee F \Leftrightarrow p$

Commutative : (i) $p \wedge q \Leftrightarrow q \wedge p$ (ii) $p \vee q \Leftrightarrow q \vee p$

Idempotent : (i) $p \vee p \Leftrightarrow p$ (ii) $p \wedge p \Leftrightarrow p$

Distributive : (i) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

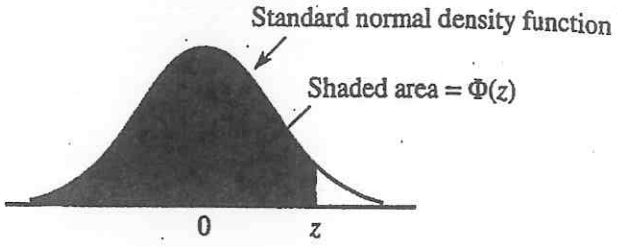
(ii) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

Associative : (i) $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r \Leftrightarrow p \vee q \vee r$

(ii) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r \Leftrightarrow p \wedge q \wedge r$

Table A.3 Standard Normal Curve Areas

$\Phi(z) = P(Z \le z)$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(continued)

Table A.3 Standard Normal Curve Areas *(cont.)*

$\Phi(z) = P(Z \leq z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998